Q1)

① $x(t) = 10\sin 40\pi t$

$c(t) = 30\cos(3\pi \times 10^3 t)$

$H(f) = X(f) \circledast C(f)$

ⓐ $m(t) = 10\sin 40\pi t * 30\cos(3\pi \times 10^3 t)$

$G_m(f) =$

$= 300\left(\frac{e^{-j40\pi t} - e^{j40\pi t}}{2j}\right) * \cos(3\pi \times 10^3 t)$

$M(f) = \left(\frac{1}{2}X(f-f_c) + \frac{1}{2}(X(f+f_c)\right)$

$= 150\left(\frac{1}{2}\delta(f-20-1500) + \frac{1}{2}\delta(f+20-1500) + \frac{1}{2}\delta(f+20+1500) + \frac{1}{2}\delta(f-20+1500)\right) * 300$

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Q1)

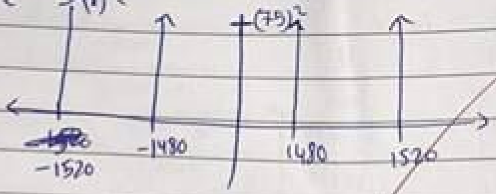
$x(t) = 10\sin 40\pi t \Rightarrow \frac{5}{2j}\delta(f-20) + \frac{5}{2j}\delta(f+20)$

$c(t) = 30\cos(3000\pi t)$

$$M(f) = \frac{30}{2}X(f-f_c) + \frac{30}{2}X(f+f_c)$$

$$= 15\left(\frac{10}{2j}\delta(f-20-1500) + \frac{10}{2j}\delta(f+20-1500)\right) + \frac{10}{2j}\left(5\delta(f+20+1500) + 5\,\delta(f+20+1500)\right)$$

$$M(f) = \frac{75}{(j)}\left(\delta(f-1520) + \delta(f-1480) + \delta(f+1480) + \delta(f+1520)\right)$$

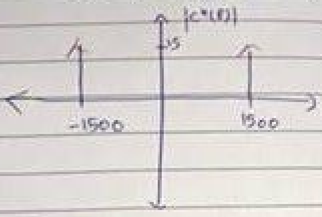


$|M(f)|^2 = G_x(f) \circledast |C(f)|^2$ $\qquad$ $G_x(f) = |X(f)|^2 = \cancel{25 \cdot 25 = 50} = 5^2 = 25$

$25 \times (15)^2 = 25 \times 15 \times 15$

(b) $c^h(t) = c(t) * \frac{1}{\pi t}$

$C^h(f) = 30\left(\frac{1}{2}\delta(f-1500) + \frac{1}{2}\delta(f+1500)\right) \cdot -j\,sign(f)$



Q2) $y'' + y' + 3y = 6x + 2x'$

① $((j2\pi f)^2 + j2\pi f + 3)\,Y(f) = 6X(f) + j2\pi f\,X(f)$

$$H(f) = \frac{Y(f)}{X(f)} = \frac{6 + j2\pi f}{(j2\pi f)^2 + j2\pi f + 3}$$

② $|H(f)| = \frac{\sqrt{36 + (2\pi f)^2}}{\sqrt{(3-(2\pi f)^2)^2 + (2\pi f)^2}}$ ⇒ not constant so it's with distortion (Amplitude)

$\angle H(f) = \tan^{-1}\left(\frac{2\pi f}{6}\right) - \tan^{-1}\left(\frac{2\pi f}{3-(2\pi f)^2}\right)$ ⇒ not linear so it's with phase distortion

③ $y(t) = |X(f)|\,|H(f)|\,e^{j(\omega t + \angle X(f) + \angle H(f))}$

$$y(t) = 10 \cdot \frac{\sqrt{36+(2\pi f)^2}}{\sqrt{(3-(2\pi f)^2)^2 + (2\pi f)^2}}\, e^{j\left(200t + \frac{\pi}{3} + \left(\tan^{-1}\left(\frac{\pi f}{3}\right) - \tan^{-1}\left(\frac{2\pi f}{3-(2\pi f)^2}\right)\right)\right)}$$

$$f = \frac{200}{2\pi}$$

Q4)

① $r(t+3)\big(u(3-t)-u(t)\big) + -r(t)\big(u(t)-u(t-3)\big)$

$$x(t) = r(t+3)\,\Pi\left(\frac{t+1.5}{3}\right) - r(t)\,\Pi\left(\frac{t-1.5}{3}\right)$$

$$x(t) = \sum_{n=-\infty}^{\infty} r(t+3-nT_0)\,\Pi\left(\frac{t+1.5-nT_0}{3}\right) - r(t-nT_0)\,\Pi\left(\frac{t-1.5-nT_0}{3}\right)$$

$$\underline{\underline{T_0 = 6}}$$

② Half wave odd symmetry $\to X_n = 0$ for n: even.

$X_n = \frac{1}{\ } \int x(t)\, e^{-jn\omega_0 t}\, dt$

$x(t) = r(t-6)\,\Pi\left(\frac{t-6}{2}\right) + \Pi\left(\frac{t+8}{4}\right)$

$X(f) = F(r(t-6)) \circledast F\left(\Pi\left(\frac{t-6}{2}\right)\right) + 4\,\mathrm{sinc}(4f)\,e^{j16\pi f}$

$F\left(\Pi\left(\frac{t-6}{2}\right)\right) = 2\sin(2f)\,e^{-j12\pi f}$

$F(r(t-6)) = \int_6^{\infty} t\,e^{-j2\pi f t}\,dt$

$u = t \qquad dv = e^{-j2\pi f t}$

$du = dt \qquad v = \frac{e^{-j2\pi f t}}{-j2\pi f}$

$\frac{jt}{2\pi f} e^{-j2\pi f t}\Big|_6^{\infty} - \frac{e^{-j2\pi f t}}{(j2\pi f)^2}\Big|_6^{\infty}$

$= 0 - \frac{j6}{2\pi f} e^{-j12\pi f} + \frac{e^{-j12\pi f}}{(j2\pi f)^2}$

$= \frac{3}{\pi f} e^{-j12\pi f} - \frac{e^{-j12\pi f}}{(2\pi f)^2}$

$F\left(r(t-6)\,\Pi\left(\frac{t-6}{2}\right)\right) = \int_{-\infty}^{\infty} 2\,\mathrm{sinc}(2(f-\lambda))\,e^{-j12\pi(f-\lambda)} \left(\frac{3}{\pi\lambda} e^{-j12\pi\lambda} - \frac{e^{-j12\pi\lambda}}{(2\pi\lambda)^2} \right) d\lambda$

$X(f) = \partial + 4\,\mathrm{sinc}(4f)\,e^{j16\pi f}$

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$$a_n = \frac{2}{T_0}\int_{-3}^{0}(t+3)\cos n w_0 t \, dt$$

$$u = t+3 \qquad dv = \cos n w_0 t$$
$$du = dt \qquad v = \frac{\sin n w_0 t}{n w_0}$$

$$\frac{(t+3)}{n w_0}\sin n w_0 t \Big|_{-3}^{0} - \frac{\cos n w_0 t}{(n w_0)^2}\Big|_{-3}^{0}$$

$$\cancel{\frac{3}{n w_0}\sin(0) + 0} - \left(\frac{\cos 0 - \cos(-3 n w_0)}{(n w_0)^2}\right)$$

$$a_n = \left(\frac{-1 + \cos(3 n w_0)}{(n w_0)^2}\right)\frac{2}{T_0}$$

n: odd

$$w_0 = \frac{2\pi}{6} = \frac{\pi}{3}$$

$$b_n = \frac{2}{T_0}\int_{0}^{3} -t \sin n w_0 t \, dt$$

$$u = -t \qquad dv = \sin n w_0 t$$
$$du = -dt \qquad v = \frac{-\cos n w_0 t}{n w_0}$$

$$\frac{t \cos n w_0 t}{n w_0}\Big|_{0}^{3} - \frac{\sin n w_0 t}{(n w_0)^2}\Big|_{0}^{3}$$

$$\left(\frac{3\cos 3 n w_0}{n w_0} - \frac{\sin 3 n w_0}{(n w_0)^2}\right)\frac{2}{T_0}$$

For a General directly from x_n

$a_n = 2\,\mathrm{Re}(x_n)$

$b_n = -2\,\mathrm{Im}(x_n)$

$$x_n = \frac{a_n}{2} + \frac{b_n}{2j}$$

$$\left(\frac{\cos 3 n w_0 - 1}{(n w_0)^2}\right)\frac{1}{T_0} + Bj\left(\frac{3\cos 3 n w_0}{n w_0} - \frac{\sin 3 n w_0}{(n w_0)^2}\right)\frac{1}{T_0}$$

$|X_n|^2$ for $n=1$ $= \frac{1}{6^2}\left(\frac{\cos\pi - 1}{(\frac{\pi}{3})^2}\right)^2 + \frac{1}{6^2}\left(\frac{9\cos\pi}{\pi}\right)^2 - \cancel{\sin\pi}$

$|X_n|^2$ for $n=2 \Rightarrow 0$

$|X_n|^2$ for $n=3 = \frac{1}{6^2}\left(\frac{\cos 3\pi - 1}{(\frac{\pi}{3})^2}\right)^2 + \frac{1}{6^2}\left(\frac{3\cos 3\pi}{\pi}\right)^2 - \cancel{\sin}$

$|X_n|^2$ for $n=4=0$

$$P_{av} = \sum |X_n|^2 = \frac{1}{6^2}\left(\left(\frac{-2}{\frac{\pi}{3}}\right)^2 + \frac{9^2}{\pi^2} + \left(\frac{-2}{\pi}\right)^2 + \left(\frac{-3}{\pi}\right)^2\right)$$